

# **MULTI-SCALE/MULTI-FUNCTIONAL PROBABILISTIC COMPOSITE FATIGUE**

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## **ABSTRACT**

A multi-level (multi-scale/multi-functional) evaluation is demonstrated by applying it to three different sample problems. These problems include the probabilistic evaluation of a space shuttle main engine blade, an engine rotor and an aircraft wing. The results demonstrate that the blade will fail at the highest probability path, the engine two-stage rotor will fail by fracture at the rim and the aircraft wing will fail at  $10^9$  fatigue cycles with a probability of 0.9967.

**KEY WORDS:** Modeling Methodology, Degradation, Carbon Fiber Composites

## **1. INTRODUCTION**

Fatigue life of structural components is a combination of local material characteristics and structural component configuration. The local material characteristics include grain structures, grain boundaries, grain size and attendant strengths, fracture toughness, etc. These in turn are very much controlled by the specific fabrication process that was used to make the material as well as the manufacturing procedure that was used to manufacture the structural component – multi-functional. The structural configuration of the component includes the geometric shape, the supports and the loading conditions (mechanical, thermal, dynamic, etc.). The variables used to describe both the material characteristics and the structured configuration do not have single values, but some scatter about mean values and certainly need multi-scale mechanics formulations. Most of the multi-scale mechanics formulations used to predict fatigue life (damage initiation, propagation, slow growth, and unstable state and/or fast growth) assumes single values for all participating variables. However, fatigue life data shows considerable scatter. The scatter is usually bounded predicatively by author-favorite method and is usually curve-fitted with the data scatter. It is very desirable, therefore, to have a multi-scale predictive method that can bind the data based on the scatter ranges in the fundamental different scales variables and the structural components variables that were mentioned previously. The objective of the proposed paper is to describe one multi-level method (multi-scale/multi-functional) that has been used with some success to a priori predict fatigue life of structural components. The method is a judicious combination of multi-scale composite mechanics, finite element structural analysis, fracture mechanics concepts, probabilistic concepts and efficient computational algorithms. The multi-scale composite mechanics in particular include a multi-factor interaction model that describes the material degradation from attendant service environment factors. In essence, the multi-level method is a computational simulation procedure (multi-scale/multi-functional) that is illustrated by applying it to an engine blade, to an engine two-stage rotor, and

to an aircraft wing. Results show that the blade will fracture along a most probable fracture path, the rotor will fracture due to multiple fracture modes, and a build-up aircraft wing will fail at  $10^9$  fatigue cycles with a probability of 0.9967.

## 2. FUNDAMENTAL CONSIDERATION

Every structure does not fail instantaneously. It fails initially gradually and then it speeds up to its final collapse state. In a computational simulation scheme each one of this type of behavior is characterized by time scales. There is a time scale where the structure may remain intact for several hours. The second stage is when the structure exhibits observable initial damage. A different time scale characterized the initial linear growth. Another one characterizes the constant growth and finally the last one which characterizes the rapid growth to fracture (Fig. 1). The truly predictive computational simulation scheme should be able to transcend all the above time scales continuously from the smallest (material local behavior, non-observable) to the largest scale (structural) which is definitely observable. What that approach will then require is to combine the material behavior through all the structure (multi-functional) and through all the different scales (multi-scale). Composite structures subjected to fatigue inherently contain both of these (multi-level). A composite structure is naturally multi-scale because of its angle ply configuration and at the same time is a multi-functional since each ply may be from different constituents (fibers/matrices) different fiber volume ration as well as different void volume ratio. Similar comments can be made of metallic structures subjected to different failure criteria. The subsequent discussion describes the probabilistic simulation of two metallic structures and one composite built-up aircraft structure.

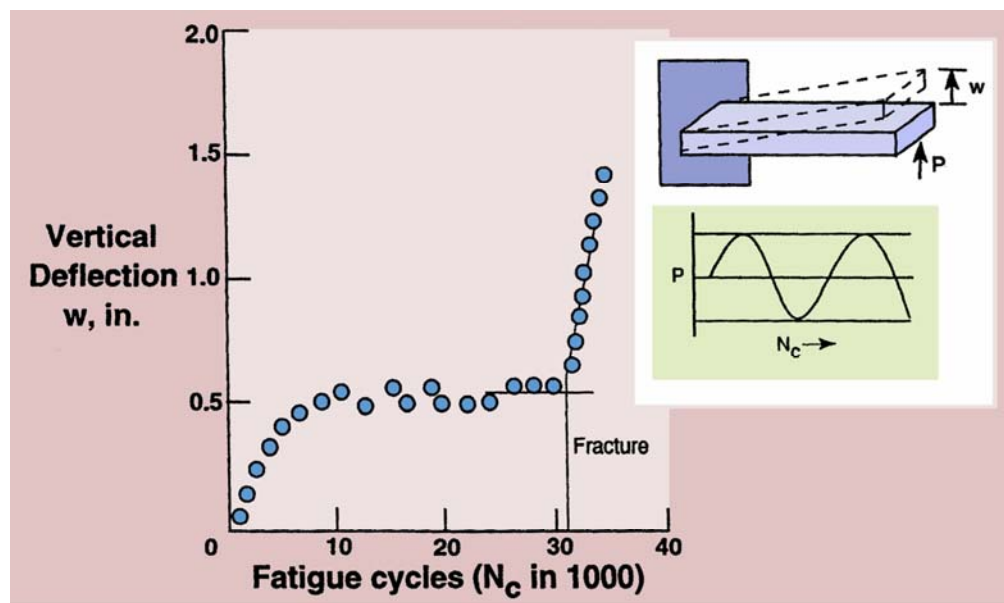


Figure 1.—Progressive fracture under cyclic load (1 in. = 2.54 cm).  
(Experimental data: Mandel, et al.).

### 3. MULTI-FACTOR SIMULATION MODEL

The multi-factors that influence a metered behavior are represented by a multifactor interaction model of multiplicative form where each factor is expressed in expanded form, as shown in Figures 2 and 3. The exponents are selected so that they satisfy the initial and final conditions, Reference 1. The exponent can have any value which describes the factor behavior from its initial value to its final value as shown in Figure 4. There are two restrictions in selecting exponents: One is that they only can take positive values and the second is that the factor within the parenthesis must have absolute value. The material degradation behavior of both the metallic structures and the composite aircraft component structure are characterized by the multi-factor interaction model.

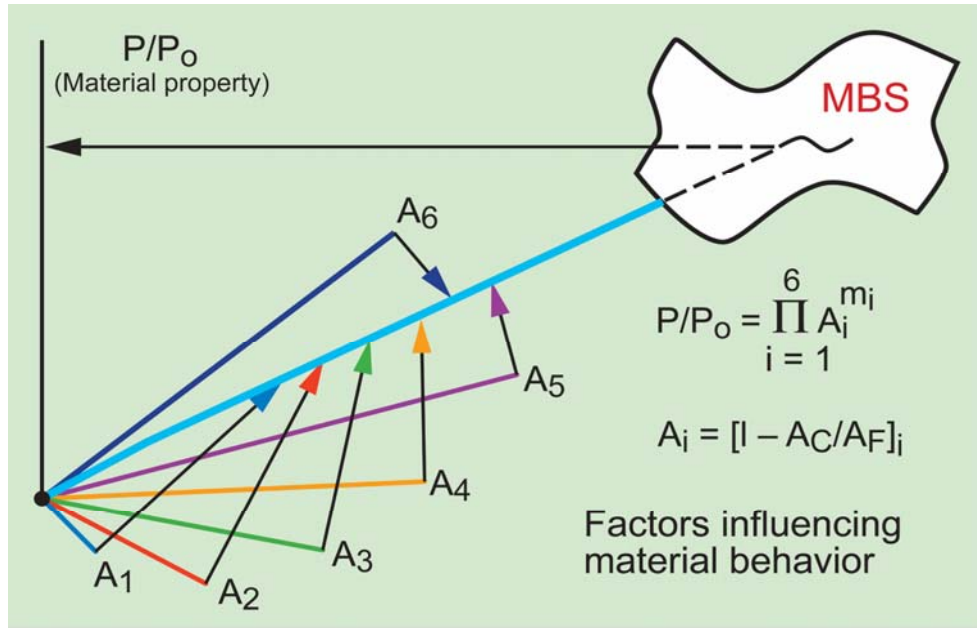


Figure 2.—Multifactor interaction model (MFIM) with substructuring.

$$\frac{M_p}{M_{p_o}} = \left( \frac{T_{gw} - T}{T_{gw} - T_o} \right)^m \left( 1 - \frac{\sigma}{S_f} \right)^n \left( 1 - \frac{\sigma t}{S_f t_f} \right)^p \left( 1 - \frac{\sigma_M N_M}{S_f N_{fM}} \right)^q \left( 1 - \frac{\sigma_T N_T}{S_f N_{fT}} \right)^r \left( 1 - \frac{\omega}{\omega_f} \right)^s \dots$$

where:

M - property	.... $\left( 1 - \frac{E_e}{E_{ef}} \right)^u \left( 1 - \frac{C_e}{C_{ef}} \right)^v \left( 1 - \frac{C_c}{C_{cf}} \right)^w \dots$
T - temperature	$E_e$ - erosion effect
S - strength	$C_e$ - corrosion effect
$\sigma$ - stress	$C_c$ - chemical/metallurgical effect
N - number of cycles	
t - time	
$\omega$ - load frequency	
m - w - are exponents	

Subscripts:

o - reference condition - assumed nominal at ambient conditions
f - final condition
M - mechanical load
T - thermal cyclic load

Figure 3.—Time dependent multifactor interaction mode (MFIM).

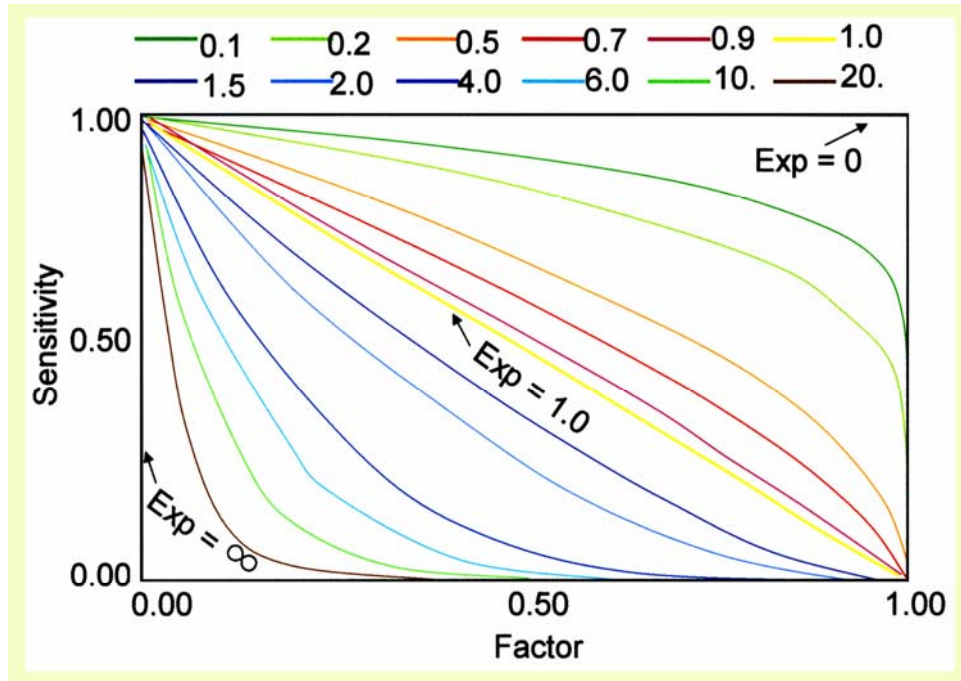


Figure 4.—Multifactor interaction model is very sensitive to factor exponent.

#### 4. PROBABILISTIC COMPUTATIONAL SIMULATION

The probabilistic simulation (Reference 2) cycle of a multilayer composite structure (multi-scale) is graphically illustrated in Figure 5. The deterministic equations to perform the simulation depicted in Figure 5 are summarized in Figure 6. Results obtained for a deterministic simulation are shown in Figure 7. Figure 7 is very important because it illustrates all the multi-functional, multi-scale (multi-level) simulations described above and it is also similar in shape to Figure 1.

#### 5. MULTI-FUNCTIONAL/MULTI-SCALE SIMULATION OF TURBINE BLADE

A space shuttle main engine blade which demonstrates a multi-level simulation is shown schematically in Figure 8. The blade is subjected to variable pressures and temperatures and rotational load as illustrated in Figure 9. The probabilistic damage propagation path which has the largest probability (0.0002) of occurring is illustrated in Figure 10. The frequency degradation along this path is shown graphically in Figure 11. Each portion in this figure illustrates the degradation in each of the four first frequencies for a total of four frequencies. This figure is very important because it graphically illustrates the damage degradation which occurs in each frequency with its respective probability. For example, the first frequency will degrade at the mean from about 5500 to 1500 cycles per second (cps) as it approached fracture. The degradation qualifies for on board health monitoring with such dramatic drop. The fourth frequency with substantial lower amplitude will degrade at the mean from about 15,000 to 3,000 cps. This is a much greater decrease. However, the instrumentation required to measure these low amplitude frequencies may be limited.

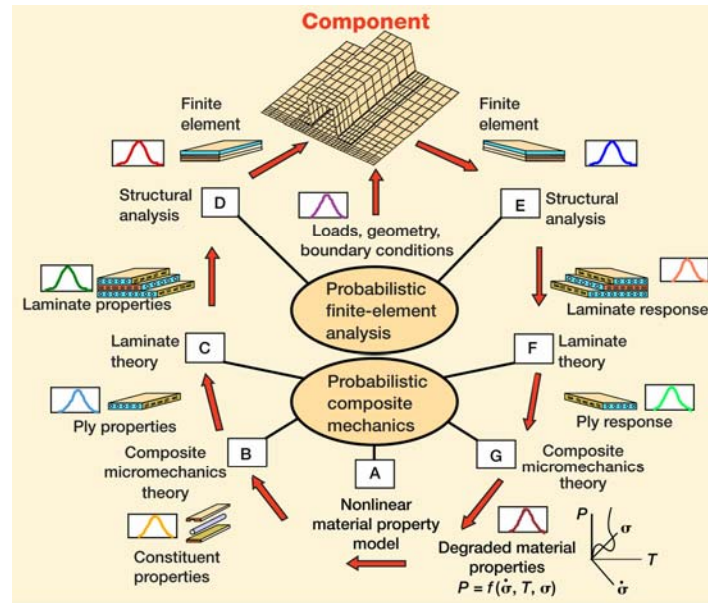


Figure 5.—Probabilistic conceptual simulation cycle (IPACS).

$$[M] \{\ddot{u}\} + [C] \{\dot{u}\} + [K] \{u\} = \{F(t)\}$$

$$\{\omega\} \leq \{\omega_a\}$$

$$\langle [K] - \omega^2 [M] \rangle \{u\} = 0 \rightarrow \{\omega\}$$

$$\{u\} \leq \{u_a\}$$

$$\{\sigma\} = [E]^{-1} [L(G)] \{u\} \leq \{S_a\}$$

$$\leq \{S_{cr}\}$$

$$\langle [K] - \lambda [I] \rangle \{u\} = \{0\} \rightarrow \{S_{cr}\}$$

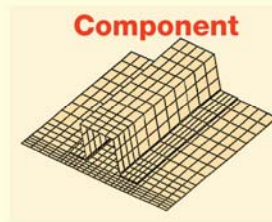


Figure 6.—Structural behavior/response governing equations.



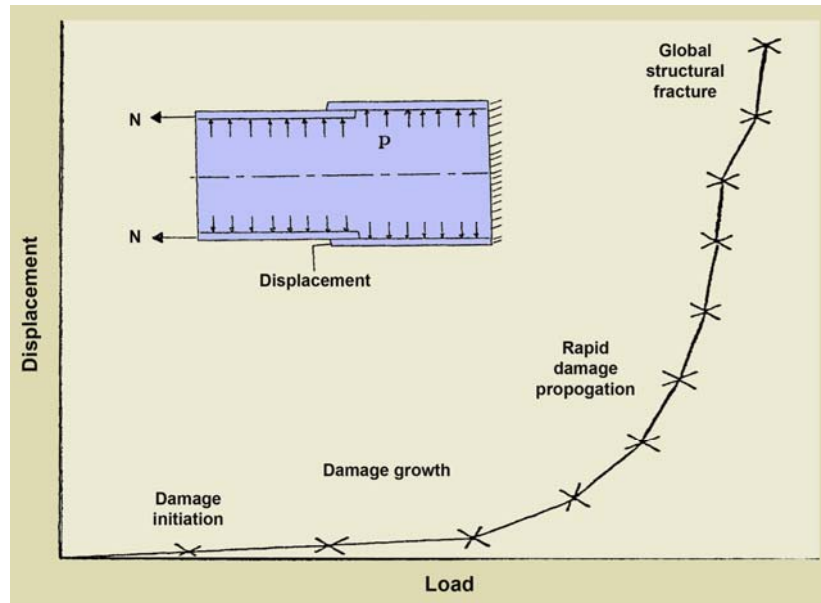


Figure 7.—Overall CODSTRAN simulation.

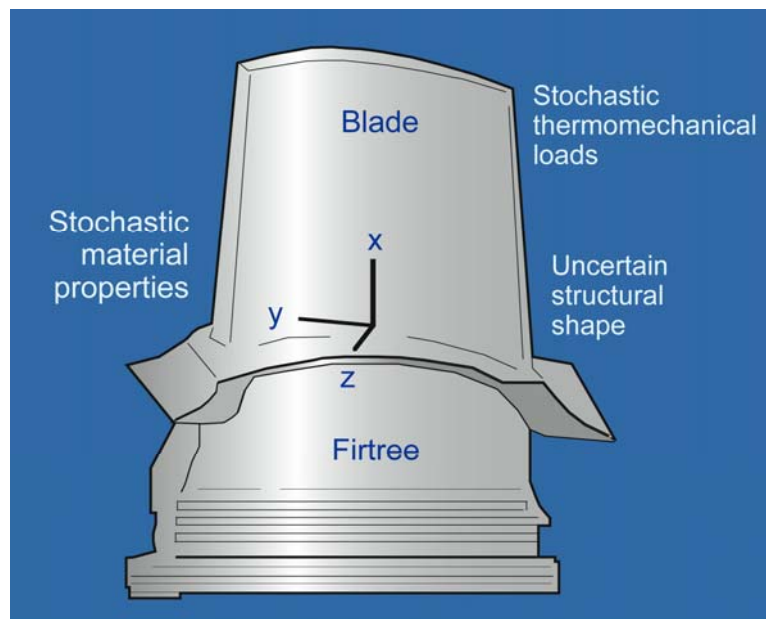


Figure 8.—A typical turbine blade.

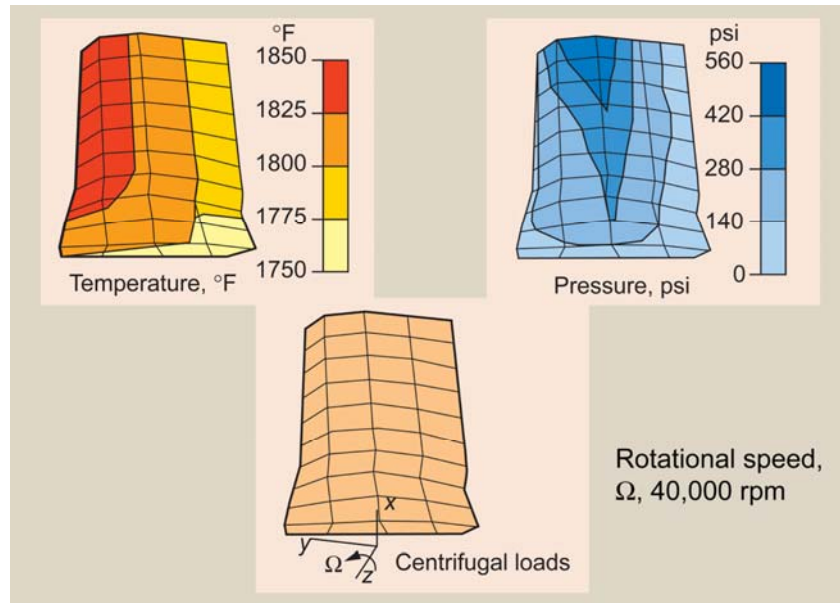


Figure 9.—Thermal mechanical loads on (SSME) blade.

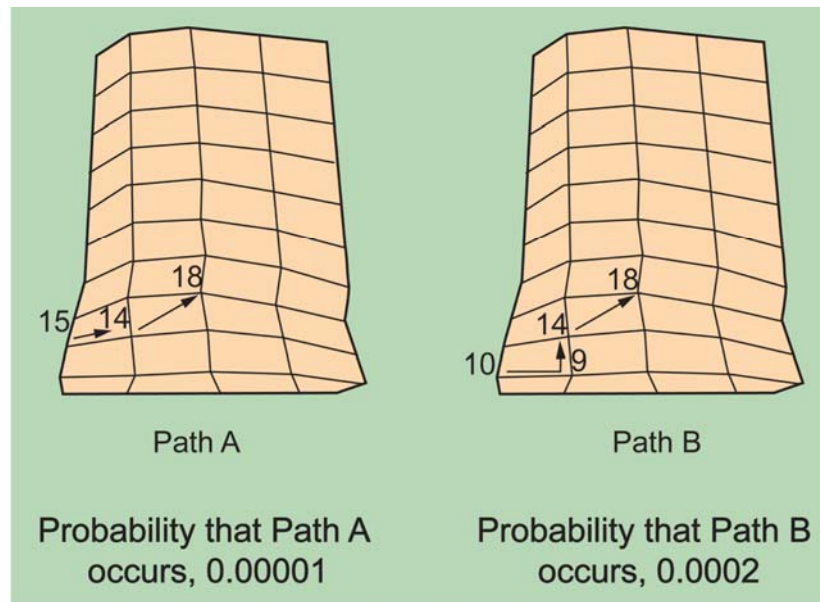


Figure 10.—Probability of component damage propagation path caused by 100,000 fatigue cycles.

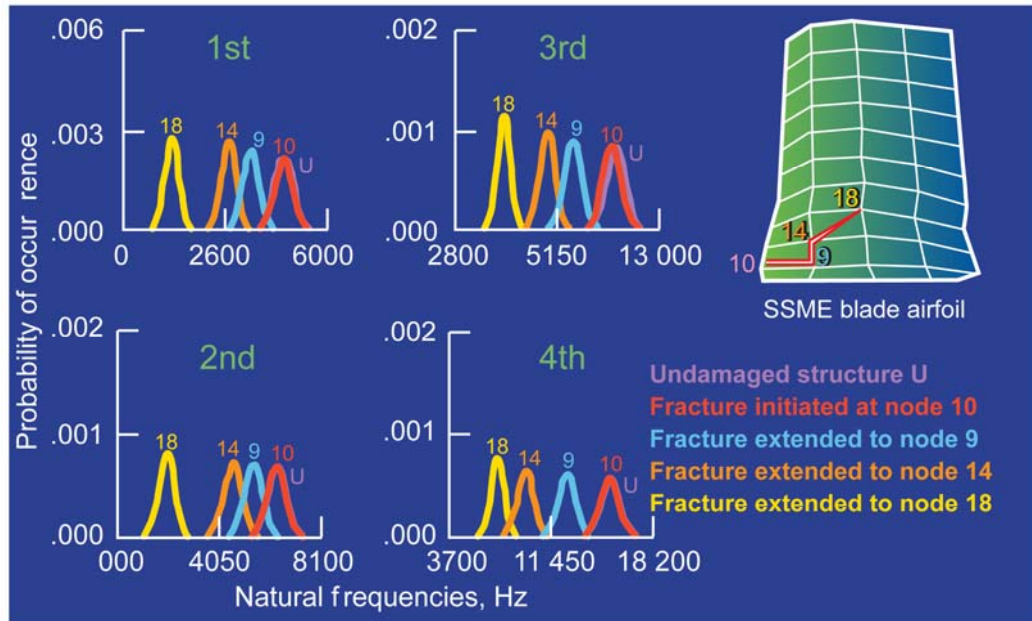


Figure 11.—Natural frequencies decreases as fracture progresses.

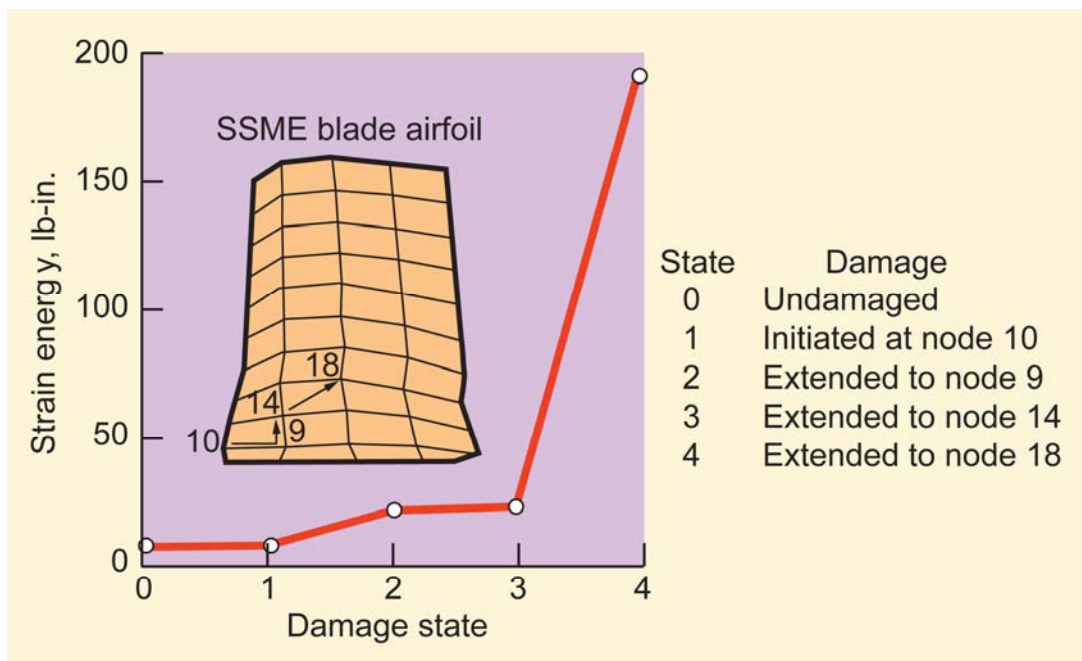


Figure 12.—Strain energy increases as the damage progresses.

Another important measure in the highest probability path is the strain energy release rate as the damage progresses from no damage to its final value. This plot is illustrated in Figure 12. This plot is another important result because illustrates probabilistic fracture toughness of the material from which the blade is made. For example, the last point before the damage starts increasing very rapidly is point 3 which corresponds to finite element node number 14. The fracture toughness is measured at the instant the material damage starts to grow very rapidly, the measurement of which is very intricate. In this simulation the fracture toughness is about



20 lb.-in. and a critical corresponding length of about one-half of the blade width. The very important observation in this discussion is that the fracture toughness is obtained as a by-product of the analysis without usage of intricate instrumentations, specimen preparation and careful measurement. In addition, the fracture path is evaluated probabilistically.

The second metallic example is two adjacent discs as is illustrated in Figure 13. The probabilistic simulation of the example requires both multi-functional and multi-scale. Multi-functional because of the different analyses included in the bottom of the figure and the multi-scale because of the different components. As is seen in this figure, the fracture at the rim has the largest probability and coincides, almost, with the system failure probability. This fracture mode is that of the rim which holds the blades. Note that the fractures evaluated are identified at the bottom of the figure. They include: (1) disc burst, (2) fracture at the bore, (3) fracture at the rim, and (4) progressive damage of the ring in yield. The ring ties the two discs. Note that the insert to the right of the figure represents the survival probability as a function of the exhausted resistance. The multi-scale is illustrated in Figure 14 which shows the probabilistic sensitivities at failure listed from the highest to the lowest. This figure is instructive in that it illustrates that the fracture toughness parameters  $RKIC$  right toughness,  $K_t$  stress concentration,  $A_0$  initial damage size,  $A$ -LCF low cycle fatigue and  $N_1$  number of cycles are relatively low valued varying from  $0.0731$  to about  $810^{-6}$ , respectively. What can we learn from these sensitivities is that fracture toughness has relatively low significance as compared to other failure modes in the design of engine disc components.

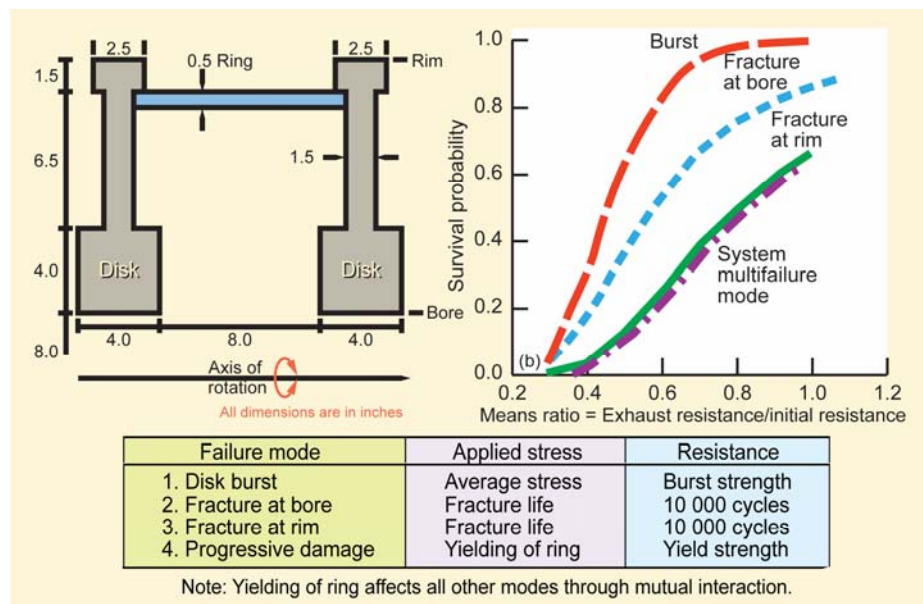


Figure 13.—Rotor system survival probability under multiple failure modes (1 in. = 2.54 cm).

Name	In u(I) Space
Speed	0.850827
Rotor dens	0.438499
Tempe	0.170793
C	0.133702
Ringy	0.073086
RK1C	0.061872
Kt	0.060917
AO	0.057976
E_ROT	0.016011
Burst	0.011983
A_LCF	0.005132
E_RIN	0.002698
Ring dens	0.000386
NI	0.000008
Toler	0.000000

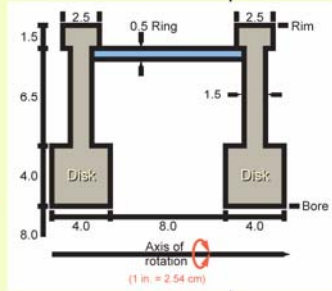


Figure 14.—Sensitivity factors of rotor system failure probability.

## 6. COMPOSITE AIRCRAFT WING

The last example of multi-level probabilistic evaluation is a simulated composite wing. The multi-scale of this example is illustrated in Figure 15 which shows the span-wise construction and in Figure 16 which shows the loading conditions (multi-functional). It is observed in Figure 15 that the wing is 5.49 m (18 ft) long by a tapering width from 1.83 m (6 ft) at the root to 1.22 m (4 ft) at the tip. The sections in this figure indicate that the internal wing structure consists of three longitudinal spans and five bulkheads. The multi-scale simulation is represented by a finite element model whereas the multi-functional model is represented by the composite mechanics which describe the thermal and hygral properties in the composite at each one of its scales.

The probabilistic results of the multi-level evaluation are shown in Figure 17 which depicts both the cumulative and the density probabilistic functions. The probability density function indicates that the mean value of this combined set of multi-functional evaluation is about  $8.5 \times 10^6$  cycles while the cumulative distribution shows that the wing will fail at about  $9 \times 10^6$  fatigue cycle with a probability of 0.9967. This evaluation demonstrates that both the multi-functional and multi-scale composite problems can be evaluated by a judicious combination of composite mechanics, finite element analysis and expedient probabilistic algorithms. The probabilistic sensitivity factors of this multi-level problem are depicted in Figure 18. The figure summarizes these factors that have a relative significance of greater than 0.1. As is indicated in this figure, the greatest significance is the cyclic loading and the fiber volume ratio and the fiber modulus, and matrix shear strength of the skin followed by the fiber modulus and the fiber volume ration of the frame, the matrix tensile strength of the skin and the void volume ratio. The significance of this evaluation is that the multitude of information that comes out of a probabilistic evaluation. Note that in the sensitivity information the multi-functional evaluation dominates the results.

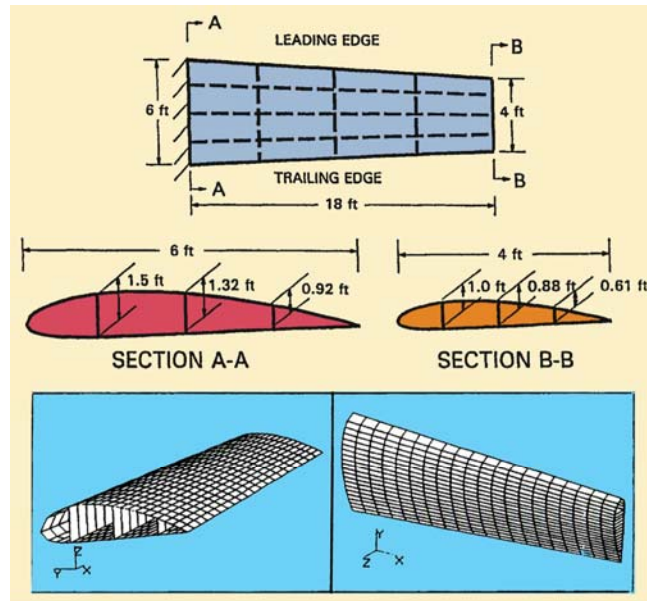


Figure 15.—Geometry and finite element model of a composite wing (1 ft = 0.305 m).

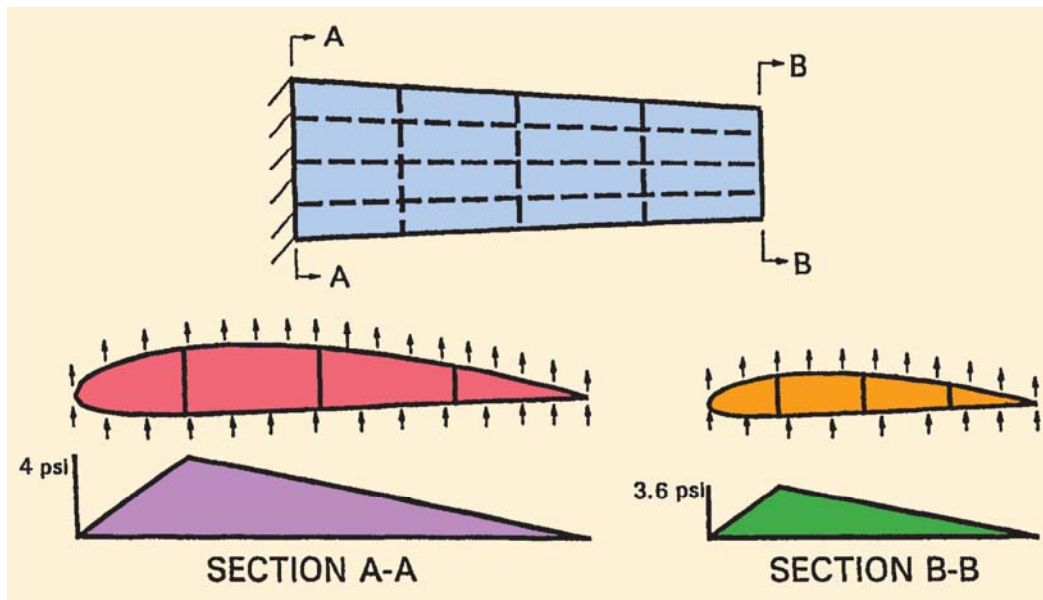


Figure 16.—Mean pressure variation on a composite wing (1 psi = 6.98 pa).

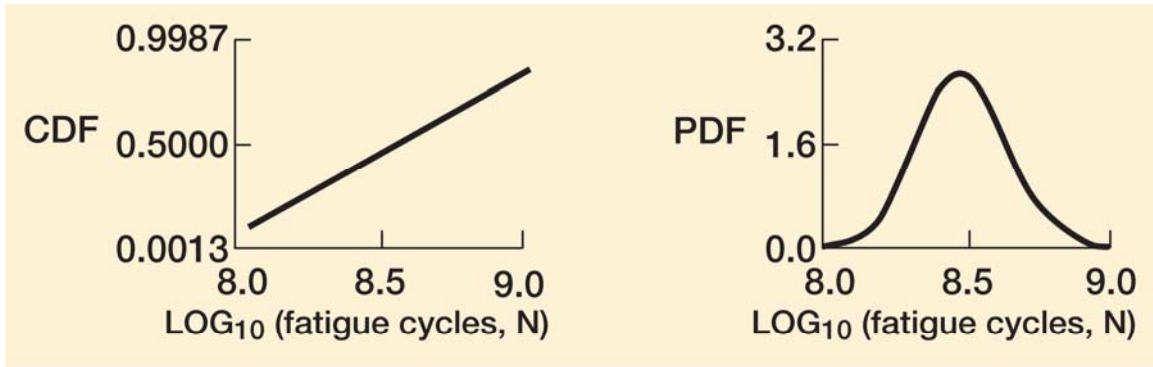


Figure 17.—Cumulative distribution function (CDF) and probability density function (PDF) of the compressive fatigue life of the composite wing for Case 5 (200 °F, 1% moisture).

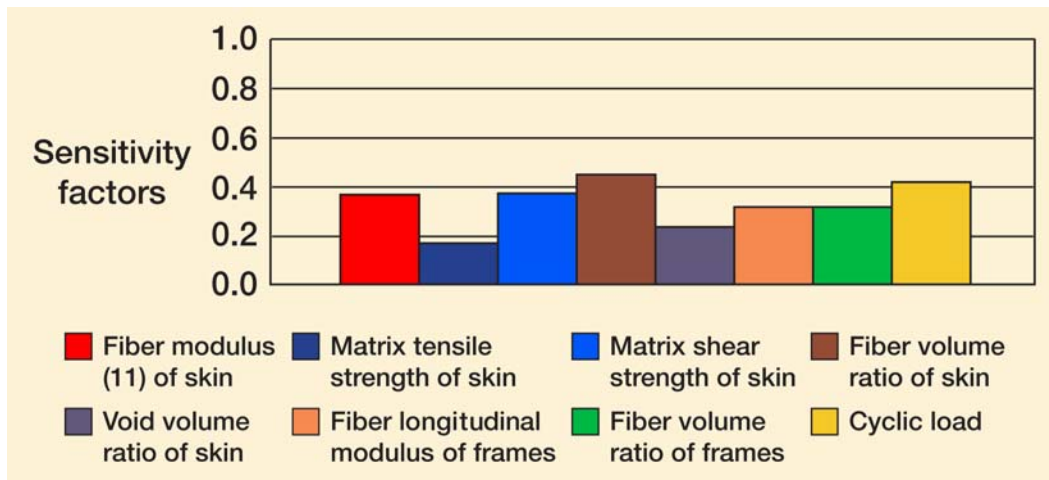


Figure 18.—Sensitivity factors of probabilistic compressive fatigue life of the composite wing at 0.001 cumulative probability for case with 200 °F temperature and 1% moisture.

## 7. SUMMARY COMMENTS

Probabilistic composite/metal structures are simulated by a multi-level simulation scheme. The multi-level evaluation includes all factors that influence component/system reliability. Multi-level is evaluated by the cumulative distribution function of the system response. Another important aspect of probabilistic multi-level evaluations is the probability sensitivities of all the variables that constitute the system design. The sensitivities are important because they are used to fine-tune the design and/or optimization variables in optimization evaluations. Probability multi-level evaluation coupled with progressive structural fracture is general for any multi-level materials and structure. Probabilistic multi-level fracture determines safe-life and fail-safe, the damage tolerance of the structure with quantifiable probability of occurrence.

## 8. REFERENCES

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